

Homework 1

Math 121A, Fall 2025.

Due: Wednesday, Sep 3, 11:59 PM

Unless otherwise stated, all solutions require clear, well-written justification. You must explicitly credit any collaborators for each problem. Problems are taken from Boas.

Problem 1. Section 1.1

- (a) Problem 12
- (b) Problem 15
- (c) Problem 16

Problem 2. In the following problems, find the limit of the given sequence as $n \rightarrow \infty$.

- (a) $\frac{(n+1)^2}{\sqrt{3+5n^2+4n^4}}$.
- (b) $\frac{2^n}{n^2}$.
- (c) $\frac{n^2}{2^n}$.
- (d) $\frac{e^n}{n!}$ (*Hint: expand out these expressions. When $n > 2e$, how is the n^{th} term related to the $(n-1)^{\text{th}}$?*)
- (e) $\frac{n^n}{n!}$.
- (f) $(1 + n^2)^{1/\log n}$.
- (g) $\frac{(n!)^2}{(2n)!}$.
- (h) $n \cos(1/n)$.

Problem 3. Use the integral test to prove the so-called “ p -series test”. The series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ is } \begin{cases} \text{convergent} & \text{if } p > 1 \\ \text{divergent} & \text{if } p \leq 1 \end{cases}.$$

Caution: do $p = 1$ separately.

Problem 4. Decide whether each of the following series are convergent or divergent.

- (a) $\sum_{n=1}^{\infty} \frac{2^n}{n}$.
- (b) $\sum_{n=1}^{\infty} \frac{2^n}{(2+\epsilon)^n}$, where $\epsilon > 0$ is a fixed positive number.
- (c) $\sum_{n=1}^{\infty} \frac{1}{\log n}$.
- (d) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$.
- (e) $\sum_{n=1}^{\infty} \frac{n-a}{n^{2b+1}+c}$, where $a, c \in \mathbb{R}$ and $b > 1$.

- (f) $\sum_{n=1}^{\infty} e^{-n^2}$. (*Hint: compare with another series, e.g. $\sum e^{-n}$.*)
- (g) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$.
- (h) $\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$.
- (i) $\sum_{n=1}^{\infty} \frac{\sin|n|}{n^2}$.

Problem 5. (*A quick survey*)

- (a) What is your preferred name and pronouns? Your major?
- (b) What do you hope to get out of this course?
- (c) Take a look at the schedule. Is there any specific content you're excited to learn about?
- (d) Take a look at the syllabus. Is there anything you would like me to clarify?
- (e) Is there anything you'd like me to know? (concerns, fun facts...)